Scholarships or student loans? Subsidizing higher education in the presence of moral hazard.*

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Abstract

An income-contingent loan scheme can at best replicate the allocation brought about by a scholarship scheme financed by a graduate tax, and only on condition that there is nothing to stop the policy maker from using tuition fees as if they were taxes. If that is not possible, even the best loan scheme will exclude some well-qualified school leaver from university. Even if individual study effort is observable, but more so if it is not, it is not socially desirable that all students should specialize in the subjects that promise the highest graduate earnings.

Key words: universities, degree choice, study effort, moral hazard, scholarships, income-contingent loans, graduate tax, tuition fees, higher-education externality.

JEL: I21, I22, I28.

1 Introduction

Whether and in which way the government should help young people pay for a higher education is a matter of great practical and theoretical interest. The arguments used to justify government intervention in this sphere are generally three. The first is based on the assumption of a positive externality.[1] Although

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[1] One possibility is that graduate and non-graduate workers are complementary, and that university education will thus increase everybody's productivity, not just that of those who bear the cost of attending university. Another is that a higher education reduces social costs by increasing social cohesion, or reducing antisocial behaviour. Bynner and Egerton (2000) report evidence of a positive association between higher education and willingness to participate in the democratic process and community activities, egalitaran attitudes, and even good parenting.

there is some controversy over the size of this external benefit,[2] nobody seems to have doubts about its existence. Also beyond dispute is the fact that education is a major factor in economic growth.[3] If growth is deemed to be desirable, there is then an argument for improving access to university, but not necessarily for subsidizing students out of general tax revenue. For the latter to be justified, it would have to be shown that the wage differential between graduates and non-graduates (the "graduate premium") does not fully reflect the private cost of higher education. The second is a market imperfection argument. A number of persons who, in the presence of a complete system of contingent markets, would have chosen to attend university may not do so because the existing markets do not allow them to (a) borrow against future earnings, and (b) insure against the risk of either an unfavourable degree result, or an unfavourable outcome in the graduate labour market. The reasons for these market imperfections are well known. Adverse selection and moral hazard problems make it unwise for insurance companies to offer cover for scholastic and earnings risks. In the absence of insurance, private lending institutions are generally reluctant to lend without suitable collateral. Since human capital cannot be mortgaged, this implies that young people will find it difficult to borrow for educational purposes unless they, or their parents, have conventional assets to offer as collateral. The third is that poor parents are less able to pay for their children's higher education out of current income, and less likely to obtain credit and insurance from the market than rich parents are.[4] The young people excluded from university as a consequence of credit and insurance market imperfections are thus likely to come from families at the lower end of the income distribution.[5] This may be interpreted as a horizontal equity argument if we regard the household as the basic decision unit, or as an equality of opportunity argument if the decision unit is the potential student.

Granted that there is a case for public intervention, which are the policy instruments? At one extreme, we may think of a scholarship scheme, financed by general tax revenue, covering both the tuition and the maintenance costs of every young person with the right personal characteristics. The good thing

[2] The numbers one gets appear to be heavily dependent on the *a priori* assumptions one makes. For the USA, Moretti (1998) puts the effect of a one percent increase in the share of college graduates on the wages of all workers at around 1.6 percent, Acemoglu and Angrist (1999) put the excess of social over private return at less than one percent. Havemann and Wolfe (1984) produce estimates far in excess of those obtained by others, and argue that conventional methods of calculation underestimate the welfare effects of education.

[3] See, for example, Barro (1998), Bassanini and Scarpenta (2001), Jorgenson and Fraumeni (1992).

[4] While there is little doubt that credit constraints are binding for many households, the US-based evidence on the higher educational effects of these constraints is somewhat controversial. Cameron and Heckman (1998), and Carneiro and Heckman (2002), find that credit quotas play no significant role in college attendance decisions. Using a sophisticated structural model, however, Keane and Wolpin (2001) argue that this is only because college students have the opportunity to support themselves by working; parental support is thus essentially a determinant of student leisure time.

[5] Dynarski (2003) finds that withdrawing social security tuition support in the US would significantly reduce enrolment and completion by disadvantaged groups.

about this policy is that it allows risk spreading, and helps to internalize the higher-education externality (if there is one). But he distributional effect is ambiguous. By allowing bright young people to attend university irrespective of family income, the scheme does in fact achieve equality of opportunity. Depending on how progressive the general income tax is, however, the cost may fall partly on the not-so-bright children of poor families who do not qualify for a scholarship. At the other extreme, we may think of a credit scheme, again covering both tuition and maintenance costs, repayable in full by the student at market interest rates. Like a scholarship, a loan makes it possible for young people with the right personal characteristics to attend university irrespective of parental support. Unlike a scholarship, however, it lacks an insurance element. Therefore, if repayment is strictly enforced in all cases (but it is difficult to see how), the scheme will not appeal to potential students from poor families.

Real-life student support schemes tend to lie somewhere in between these two extremes. A common arrangement is to have tuition subsidized on a sliding scale according to parental income ("need"), and maintenance costs covered in some measure by either a scholarship, or a loan repayable at below market interest rates. The award of a scholarship or a subsidized loan is usually conditional on school record and university performance ("merit"). These in-between arrangements obviate some of the problems, but miss out on some of the benefits, of pure schemes. One way to obviate the possibly regressive effect of scholarships financed by general tax revenue is to have the scheme financed by an income-tax surcharge on graduates (a "graduate tax"), but the idea does not appear to have attracted much political support. An idea that is becoming increasing popular, by contrast, is that of income-contingent loans.[6] These differ from straight loans in that the size of the repayment is conditional on the amount that the borrower earns after graduation (we would argue that *all* student loans are *de facto* income-contingent, because it is difficult to enforce the repayment of a loan on an unsuccessful student). Income-contingent loans thus have both a redistributive, and an insurance element. A problem is that the scheme can break even only if successful graduates are asked to pay back, on average, more than the value of their loan capitalized at market interest rates. As participation is obviously voluntary, bright young people would then accept a loan only if they were credit constrained. As a consequence, young people rich enough to pay for a higher education out of family resources would do so, and not participate in the loan scheme. As there would then be less money to redistribute, some bright young people from poor families might be denied a loan, and consequently excluded from university.

The question whether a student support policy is justified is addressed in De Fraja (2002). Assuming a utilitarian social welfare function, the author finds that public intervention is indeed justified, and that the second-best policy

[6] The idea comes originally from Friedman (1962), but was resuscitated in Barr (1991). It has been fully implemented in Australia and subsequently New Zealand. Elements of it are present in a number of countries including the Netherlands, South Africa, and the UK. Sweden experimented with it, but then abandoned the idea. See Chapman (2006) for a discussion of different forms of income contingent loans, and of recent experience.

redistributes in favour of richer and more talented students. The choice of policy instruments is investigated in Garcia-Peñalosa and Wälde (2000), Hanushek *et al.* (2003, 2004), and Del Rey and Racionero (2006). The first of these papers compares four alternative policies: pure loans, an education subsidy financed by general tax revenue, an education subsidy financed by a tax on graduates, and income-contingent loans. Rather than maximizing a social welfare function, the authors rank these policies on the basis of three criteria: Pareto optimality, *ex ante* equality of opportunity, and *ex post* equality of lifetime income. They find that the graduate tax scheme comes out the winner on all scores. The two Hanushek *et al.* papers use a calibrated general equilibrium model. The policies considered are tuition subsidies, need-based grants, merit-based awards, and income-contingent loans. They find that education subsidies in general perform less well than other forms of redistribution where equity is concerned. If there is an education externality in production, however, the case for education subsidies becomes overwhelming. Need-based grants achieve greater equality than merit-based ones. Income-contingent loans perform rather badly. Del Rey and Racionero adopt output maximization as the optimality criterion. The policy that is optimal by this criterion fully insures the lowest-ability individual included in the scheme, and partially insures those with higher ability. All but one of these papers allow potential students to differ in their ability to learn. Del Rey and Racionero allow graduates to differ in their productivity (wage rate), but not in the probability of becoming a graduate. De Fraja, and Hanushek *et al.*, allow parents to choose how much support to give their children. Garcia-Peñalosa and Wälde, and Del Rey and Racionero, take such support as exogenous. In De Fraja, there is an adverse selection problem, arising from the assumption that a student's learning ability is known to the student, but not observable by the government. In the other papers, ability is uncertain, but there is no adverse selection problem because the government is assumed to have the same information as the student. As none of these papers allows university success to depend on individual study effort, there is no moral hazard. There is, furthermore, no choice of degree content.[7]

Like our predecessors, we start from the premise that, in the absence of a complete system of contingent markets, a number of school leavers who would have otherwise gone to university will go straight into the labour market. This implies that it may be possible to raise social welfare by helping at least some of these young persons to attend university. The welfare gain will be larger if there is a positive higher-education externality. Unlike our predecessors, however, we focus on the incentive problem that may arise from the non-observability of individual study effort, and on the subject choice problem associated with differences in individual aptitude for the study of different subjects. We postulate that:

(i) school leavers differ not only in their learning ability, but also in their

[7] The latter is examined in connection with graduate migration by Poutvaara (2004). Given the focus of that paper, the relevant distinction is between subjects that contain internationally applicable knowledge, and subjects containing knowledge that can be used only in the country of origin.

aptitude for the study of different subjects, and in the amount of support they receive from their parents;

(ii) absolute and relative learning abilities are observable by both the school leaver's parent and the policy maker;

(iii) parents derive utility not only from their own, but also from their children's present and future consumption;

(iv) parental income is observable by the policy maker, parental support is not;

(v) the degree result is a random variable, with probability distribution conditional on study effort;

(vi) graduate earnings are a random variable, with probability distribution conditional on degree result and subject mix;

(vii) the type of degree chosen by the student is observable by his parent, and by the policy maker;

(viii) study effort is observable by the student's parent, but not by the policy maker;

(ix) individuals are risk averse, but cannot buy insurance against academic and labour market risks;

(x) the policy maker can borrow, parents and children cannot.

Postulate (i) extends the literature cited by recognizing that young people may differ not only in absolute, but also in comparative learning ability. As a simplification, we shall assume that absolute ability can take only two values, high or low, and that a low-ability person would not benefit from a higher education under any circumstances. This allows us to divide school-leavers into two groups, "university material" and the rest. We similarly assume that high-ability school leavers divide into two groups, those with an aptitude for the study of subjects (dubbed "science") which offer the prospect of a well-paid career, and those with an aptitude for the study of subjects (dubbed "arts") which do not. Postulate (ii) is justified if school records are informative about a school leaver's potential for higher education, and aptitude for the study of different subjects,[8] or if admission to university is conditional on passing an informative entry examination. Postulate (iii) is the usual descending-altruism assumption. The first part of (iv) is justified if income-tax returns are truthful. Postulate (v) recognizes that the degree result (not just pass-or-fail as assumed in most of the literature, but the actual mark)[9] has a random component. Postulate (vi) adds a second layer of uncertainty to the outcome of an educational investment.[10] The first part of (viii), to be relaxed later, is easier to justify if students live at home, than if they live away. In either case, however, parents have an observational

[8] This in turn implies that primary and secondary education are sufficiently subsidized, and the school curriculum sufficiently broad, to provide such information about all children of the relevant age.

[9] In some university systems, such as the British or the Italian one, there is an actual degree result. Where this is not the case, as in the US system, we may think of the degree result as of the average of the grades obtained in individual examinations.

[10] Although graduates earn more than non graduates on average, there is a great deal of variation in expected earnings across subjects. Graduates in certain subjects earn less than some non graduates; see, for example, Blundell, Dearden, Goodman and Reed (2000).

advantage over the policy maker. The second part of the postulate gives rise to a moral hazard problem, not contemplated in any the papers cited. Postulates (ix) and (x) give the policy maker scope for raising social welfare by anticipating the benefits of educational investments, and reducing individual risk.

Our primary aim is to find the higher education policy (fee structure, and student support scheme) which will achieve the largest expected welfare gain over *laissez faire,* under the constraint that the policy will be subsidized by the general tax payer only insofar as there is a positive higher-education externality. Rather than starting with a list of policies, and asking which is preferable according to some criterion, we start by characterizing the allocation which maximizes a Benthamite social welfare function, and then look for ways to implement it. Although we are ultimately interested in the second best, we characterize also the first best, because that will allow us to distinguish equity and insurance from incentive considerations. We find that both the first and the second best can always be implemented using a scholarship scheme financed by a graduate tax. A loan scheme, even an income-contingent one, can at best replicate the allocation generated by a scholarship-*cum*-graduate-tax scheme. If tuition fees are restricted to be no higher than the average total cost of universities,[11] an income-contingent loan scheme will exclude more than the second-best number of excluded high-ability school leavers from poor families, and consequently deny them a higher education. If there is also a "no usury" constraint, such that a student cannot be required to repay an educational loan at more than the market rate of interest, then not only some poor but also some rich high-ability school leavers will be excluded from the scheme. Unlike the poor, however, the rich will not be excluded from a higher education. The number of excluded agents is a decreasing function of the educational externality.

Our secondary aim is to establish whether students should be allowed to follow their own inclination in the choice of subjects, or pushed towards the subjects which promise higher monetary rewards. We find that, in the absence of moral hazard, it would be optimal (first best) for scientifically-inclined students to specialize completely in science subjects, and artistically-inclined ones to specialize partially in arts subjects. Given moral hazard, it will still be optimal (second best) for scientifically-inclined students to specialize completely in science, but artistically-inclined ones should be induced to specialize in the arts *further* than would be efficient conditional on the policy. The latter is the only sense in which second-best policy can be said to distort educational decisions. We also find that, in *laissez faire* as in first and second best, scientifically inclined students put in more effort than artistically inclined ones.

2 Social optimization as a multi-agency problem

We shall assume that universities are publicly owned or, equivalently, that the government will make good any difference between tuition fees and tuition costs. If there is no higher-education externality, the policy must break-even. If there

[11] Possibly defined to include also the cost of doing research.

is, the policy will be subsidized out of general tax revenue by the monetary value of the externality. There are three categories of actors: high-ability school leavers, their parents, and the policy maker. High-ability school leavers are of two types, indexed $i = a,\ s$. Type a has a comparative advantage in the study of the arts, type s in that of science (recall that arts and science are just shorthand for low and high expected graduate earnings). The precise sense in which we speak of comparative advantage in the present context is explained in the next section. Parents differ in their income, and in the type of child they have.

The policy optimization has the structure of a multi-agency problem, with the policy maker in the role of principal, and the parents of high-ability school leavers in that of agents. According to the logic of agency problems, if an action is either unobservable or costly to observe, the agents must be given the incentive to undertake it at the level desired by the principal. In the present context, the actions falling into this category are study effort and parental transfers. By contrast, if an action is observable, it does not make sense for the principal to offer costly incentives, because the agents can be obliged to undertake it at the level desired by the principal by what is politely called a "forcing contract" (in plain English, by threatening the agent with a penalty high enough to dissuade him from doing otherwise). In the present context, an action falling into this category is the choice of degree type. The principal will design the policy in such a way that it is in the agent's interest to accept the help (scholarship, loan) offered to him, and then make this help conditional on the agent choosing the prescribed degree course.[12] In view of (iii), there may be also another principal-agent problem, this time with the parent in the role of principal, and the child in that of agent, but this is a reflection of the one involving the parents and the policy maker. In the absence of a public policy conditioning student loans or scholarships on degree results, there would in fact be no conflict between parent and child over the choice of degree and study effort. Even in the presence of such a policy, however, postulates (vii) and (viii) allow us to disregard this secondary principal-agent problem because the parent can always use a forcing contract (e.g., threaten to withdraw support) to keep his child in line. If we relaxed (viii), and parents were thus unable to observe their children's study effort, we would have a common-agency problem.[13] We shall briefly deal with that in the concluding section.

We consider three dates, labelled $t = 0, 1, 2$. Parents are alive at dates 0 and 1, children are alive also at date 2. The principal announces his policy at date 0. The agents take their decisions at date 1. Degree results become available between dates 1 and 2. Graduate earnings are revealed at date 2. The policy consists of a net payment from the policy maker to some of the students at

[12] Restrictions on the student's choice of degree course exist in all university systems which practice some form of selection at entry. If selection is by school results, the access to a particular type of degree is typically conditional on the student having achieved certain minimum grades in certain specified school subjects. If selection is by entry examination, the student may have to pass a test common to all degree courses, but typically also one specific to that particular type of degree.

[13] On the structure of common-agency problems see, for example, Bernheim and Whinston (1986).

date 1, and from some of the graduates to the policy maker at date 2. The former may be interpreted as the difference between a scholarship or loan and a tuition fee, the latter as a graduate tax or loan repayment. Date-2 money values are discounted back to date 1 at the market rate of interest. Irrespective of whether it is interpreted as a loan repayment or a graduate tax, we shall assume that the date-2 payment is collected at zero marginal cost,[14] and may be contingent on any of the variables that the principal observes at the relevant date. There is, however, a fundamental difference between a scholarship and a loan scheme. A student can avoid having to pay back a loan by not accepting one in the first place, but cannot escape a graduate tax by turning down the offer of a scholarship. Throughout, we shall assume that all policy instruments other those mentioned are beyond the principal's control (and express parental incomes, and graduate and non-graduate wages, as net of any general income tax).

3 Agents

At date 1, a high-ability school leaver can go either into the labour market, or to university. In either case, he will receive a nonnegative transfer m_1, not necessarily the same in the two cases, from his parent. In the first case, he will earn w at both dates. In the second, he will earn nothing at date 1, and $w + m_2$, where m_2 is the graduate premium, at date 2; furthermore, he will receive a payment y_1 from the principal at date 1, and make a payment y_2 to the principal at date 2. We can think of y_1 as the difference between a scholarship or loan denoted by χ,[15] and a tuition fee denoted by θ,

$$y_1 \equiv \chi - \theta, \quad \chi, \theta \geq 0, \tag{1}$$

and of y_2 as a graduate tax or loan repayment. Notice that y_1 can be negative, but not lower than $-\theta$. By contrast, y_2 can be only positive or zero.

The graduate premium, m_2, is a random variable distributed over the closed interval $M_2 = [0, \overline{m}_2] \in R$ with density $g(m_2|x, d)$ conditional on degree type, d, and degree result, x. We characterize the degree type by the proportion of science subjects contained in it, such that d takes value 0 if the student takes only arts subjects, 1 if he takes only science ones. If a student fails his degree, the premium will be zero. A degree with a high science content, or with a high mark, makes it more likely that the graduate will attract a large premium. More precisely, the cumulative distribution of m_2, $G(m_2|.)$, associated with a higher (x, d) first-order stochastically dominates the one associated with a lower (x, d). An increase in the science content of the degree does not affect the marginal effect of the degree result on the distribution of m_2. Using the subscripts d and

[14] In Australia, loan repayments are collected at no extra cost together with income tax; see Chapman (1997).

[15] If χ is conditional on final degree result, which will be known only at date 2, we must assume that the student receives partial advances on χ as examination results come in, and the balance when he gets the degree.

x to denote partial differentiation with respect to these variables, we can then write

$$G_d(m_2|x,d) \leq 0,\ G_x(m_2|x,d) \leq 0,\ G_{x,d}(m_2|x,d) = 0.$$

For each (x,d), there is some m_2 such that the first two of these restrictions hold as inequalities. We make the CDF assumption that $G(.)$, is convex in (x,d).[16] For the time being, we also assume that m_2 is i.i.d. (this simplifying assumption will be relaxed at the end of section 7).

The final degree result, x, is a random variable distributed over the closed interval $X = [\underline{x}, \overline{x}] \in R^+$ with density $f(x|e)$ conditional on effort, e. Effort varies in the closed interval $E = [\underline{e}, \overline{e}] \in R^+$. Studying hard raises a student's chances of obtaining a good degree result in the precise sense that the cumulative distribution of x, $F(x|.)$, associated with a higher e first-order stochastically dominates (FOSD) the one associated with a lower e,

$$F_e(x|e) \leq 0.$$

The e subscript denotes partial differentiation with respect to this variable. For each e, there will be some x such that the condition holds as an inequality. We assume that the monotone likelihood ratio (MLR), and convexity of the distribution function (CDF), conditions are satisfied.[17] For simplicity, we shall start by assuming that x is i.i.d. too.[18]

The lifetime utility of a type-i student is a random variable with expected value

$$E(U) = \int_x u_1(m_1 + y_1) f dx - z^i(e,d) + \int_x \int_{m_2} u_2(w + m_2 - y_2) g dm_2 f dx, \quad (2)$$

where $z^i(e,d)$ is the type-specific disutility of applying the level of effort e to the study of the subject mix d. The $u_t(.)$ functions are increasing and concave. Normalizing the subsistence consumption to zero, we shall assume that $u'_t(0) = \infty$. The $z^i(.)$ functions have the following properties.

1. If the student exerts no effort, the disutility is zero whatever the subject mix,

$$z^i(0,.) = 0.$$

2. For any positive effort level, the disutility of a type-a student who studies only arts is equal to that of a type-s student who studies only science,

$$z^a(e,0) = z^s(e,1),\ e > 0. \quad (3)$$

[16] Together with the MLR and CDF restrictions imposed on the distribution of x, $F(x|e)$, this standard assumption ensures the concavity of the expected utility function (see Appendix A0), and thus the uniqueness of the agent's choice of effort. This will allow us to substitute the agent's first-order conditions for the incentive-compatibility constraints in the policy optimization problem.

[17] These are restrictions on the form of $f(.)$. The former requires that (f_e/f) is increasing in x, the latter that the cumulative distribution of x is convex in e.

[18] This assumption is not crucial for our results. As we shall point out at the end of section 7, if the xs of the different students were correlated, that would only slightly modify the form of the second-best payments from and to the principal.

3. Disutility is increasing and convex in effort,

$$z_e^i(e,d) > 0,\ z_{ee}^i(e,d) > 0. \tag{4}$$

4. For a type-a student, disutility is increasing and convex in science content,

$$z_d^a(e,d) > 0,\ z_{dd}^a(e,d) > 0. \tag{5}$$

5. For a type-s student, disutility is decreasing in science content,

$$z_d^s(e,d) < 0. \tag{6}$$

6. The marginal disutility of effort is increasing in science content for an a, non-decreasing for an s,

$$z_{ed}^s(e,d) \leq 0 < z_{ed}^a(e,d), \tag{7}$$

If a high-ability school leaver goes straight into the labour market, his future is certain. His lifetime utility is given by

$$U = u_1(w+m_1) + u_2(w). \tag{8}$$

His parent's utility is

$$V = v(k-m_1) + \rho[u_1(w+m_1) + u_2(w)], \qquad 0 < \rho < 1.$$

where k is the parent's income, observable by the principal, and the function

$v(.)$ is increasing and concave. Parental income varies in the closed interval $K = [0, \overline{k}] \in R^+$. Normalizing the parent's subsistence consumption to zero, we impose $v'(0) = \infty$. The pay-off to the parent of letting the child become a worker at date 1 is then

$$\pi_W(k) = \max_{m_1} V. \tag{9}$$

In this case, m_1 satisfies the first-order condition

$$-v'(k-m_1) + \rho u_1'(w+m_1) \leq 0, \tag{10}$$

with the strict inequality holding for $m_1 = 0$.

If the child goes to university, the parent's utility is a random variable. Assuming that the parent chooses m_1 after observing y_1, his expected utility is

$$E(V) = \int_x v(k-m_1) f dx + \rho E(U), \tag{11}$$

where the argument of $E(U)$, y_1, and consequently m_1, may now depend on x.

The pay-off to the parent of letting a type-i school leaver go to university is

$$\pi_S^i(k, y_1, y_2) = \max_{(d,e,m_1)} \int_x \{v(k-m_1) +$$
$$\rho[u_1(m_1+y_1) - z^i(e,d) + \int_{m_2} u_2(w+m_2-y_2)\, g dm_2]\} f dx.$$

We show in Appendix A0 that $E(U)$ is concave, so that (d, e, m_1) satisfies the first-order conditions

$$z_d^i(e,d) = \int_x \int_{m_2} u_2(w+m_2)\, g_d dm_2 f dx, \tag{12}$$

$$z_e^i(e,d) = \int_x \left[\frac{1}{\rho} v(k-m_1) + \left(u_1(m_1+y_1) + \int_{m_2} u_2(w+m_2-y_2)\, g dm_2\right)\right] f_e dx \tag{13}$$

and

$$-v'(k-m_1) + \rho u_1'(m_1+y_1) \leq 0. \tag{14}$$

The last of these conditions must hold for each x, with the strict inequality again holding for $m_1 = 0$.

A type-i school leaver will then attend university if and only if

$$\pi_S^i(k, y_1, y_2) \geq \pi_W(k). \tag{15}$$

Let I denote his date-1 income, so that $I = w$ if he becomes a worker, and $I = y_1$ if he becomes a student. Using either (10) or (14), we may write

$$m_1 = m(k, I), \tag{16}$$

with $m_k > 0$ and $-1 < m_I < 0$ for all (k, I) such that $m_1 > 0$. Assuming $u_1''' = v''' = 0$, m_k and m_I are constant.[19]

4 Laissez faire

We define *laissez faire* as a situation where there is no student support policy (but there may be other forms of public intervention, and this is why we define k, w and m_2 as net of any general income tax, which we take as exogenous). Therefore,

$$y_1 = -\theta,\ y_2 = 0 \text{ and } \theta = p, \tag{17}$$

where p is the average total cost of a university education,[20] assumed the same for all universities. Let the LF subscript denote the *laissez-faire* value of a variable. Given that y_1 does not depend on x, the pay-off to a parent of letting his type-i child go to university is

$$\pi_{S/LF}^i(k, -p) = \max_{(d,e,m_1)} \{v(k-m_1) + \tag{18}$$

$$\rho[u_1(m_1-p) - z^i(e,d) + \int_x \int_{m_2} u_2(w+m_2-y_2)\, g dm_2 f dx]\}$$

In Appendix A1, we prove the following.

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[19] In fact, $\frac{\partial m_1}{\partial k} = \frac{v''}{v''+\rho u_1''}$ and $\frac{\partial m_1}{\partial I} = -\frac{\rho u_1''}{V''+\rho u_1''}$.

[20] Possibly inclusive of the cost of doing reasearch.

Proposition 1. *In laissez faire,*

i) there exists a threshold level of parental income, $\widetilde{k}^i_{LF}$*, below which a type-i school leaver will not attend university (the threshold is no lower for* $i = a$ *than for* $i = s$*);*

ii) all type-i students choose the same subject mix, d^i_{LF}*, and the same effort level,* e^i_{LF}*;*

iii) type-a students take more arts subjects, and supply less study effort, than type-s students (who will take only science subjects),

$$d^a_{LF} < d^s_{LF} = 1 \text{ and } e^a_{LF} < e^s_{LF}.$$

Therefore, in *laissez faire*, some school leavers may not get a higher education despite having the right personal qualities. Such a school leaver will not go to university either because he does not get enough parental support to cover the tuition fee, $m(k, -p) < p$, or because, taking into account that graduate earnings are uncertain, the expected graduate premium is not large enough to compensate for the loss of date-1 earnings. Given that the expected premium is higher for an s than for an a, the level of parental income below which it is not profitable to buy a university education is higher for an a than for an s. For the same reason, an s will take only science subjects but an a will not take just arts subjects, and an a will study less hard than an s. Note that the choice of subject mix and effort level depends only on expected gains (adequate parental support is necessary for a school leaver to attend university, and may raise his date-1 utility, but will not affect his choice of d and e). For this reason, there would be no conflict of interest between parent and child over the choice of d and e even if the latter were not observable by the parent.

5 The principal

Let j denote a generic high-ability school leaver. The principal's objective function is assumed to be the sum of those of the agents,[21]

$$\begin{aligned} W \quad = \quad & \sum_j \int_{x^j} v(k^j - m_1^j) f dx^j + \qquad (19) \\ & \sum_j \int_{x^j} \rho \left[\left(u_1 \left(m_1^j + y_1^j \right) - z^j + \int_{m_2^j} u_2 \left(w + m_2^j - y_2^j \right) g dm_2^j \right) \right] f dx^j. \end{aligned}$$

For the i.i.d assumption,[22] y_t^j can only depend on $\left(k^j, x^j, m_2^j\right)$. Since (19) is a sum of concave functions, the principal is averse to risk like his agents. Assuming

[21] Recall that the agents are the parents of high-ability school leavers. As a parent's utility is a function of the child's utility, however, we shall interchangeably talk of agents, parents or students to suit the context.

[22] This allows us to avoid writing the expected utilities of the different agents as functions of the joint density of the different outcomes; see Holmström (1982) and Mookherjee (1984).

that the number of agents is "large", however, the principal does not face any uncertainty over how much he will have to pay out in total to students at date 1, and how much he will get back in total from graduates at date 2. Using (1), and allowing for the possibility that the tuition fee charged to j, θ^j, is different from p, we can then write the principal's intertemporal budget constraint in expected-value terms as

$$\sum_j \left(\int_{x^j} \left(y_1^j + p - \int_{m_2^j} y_2^j g dm_2^j \right) f dx^j \right) \leq S, \tag{20}$$

where S is the monetary value of the higher-education externality.

As parental support and student effort are not observable by the principal, there is also an incentive-compatibility constraint for each of these variables. From (16), the constraint on m_1^j is simply

$$m_1^j = m\left(k^j, y_1^j\right). \tag{21}$$

For our assumptions on the probability distributions of x^j and m_2^j, the incentive-compatibility constraint on e^j can be replaced by the first-order condition on the agent's choice of this variable,

$$z_{e^j}^j = \int_{x^j} \left[\frac{1}{\rho} v(k^j - m_1^j) + u_1\left(m_1^j + y_1^j\right) + \int_{m_2^j} u_2\left(w + m_2^j - y_2^j\right) g dm_2^j \right] f_{e^j} dx^j. \tag{22}$$

There is no such constraint on d^j because the choice of degree type (subject mix) is observable by the principal. Finally, there are the university-participation constraints

$$\pi_S^j\left(k^j, y_1^j, y_2^j\right) \geq \pi_W(k^j). \tag{23}$$

Since (14) holds as equality if $m_1 > 0$, and $m_I = 0$ for $m_1 = 0$ (see the last line of section 3), the first-order conditions for the maximization of (19)[23] subject to (20) – (23) can be written as

$$\left[u_1'\left(m(k^j, y_1^j) + y_1^j\right)\left(1 + \nu^j\right) - \lambda\right] f + \mu^j u_1'\left(m(k^j, y_1^j) + y_1^j\right) f_{e^j} = 0, \tag{24}$$

$$\left[-u_2'\left(w + m_2^j - y_2^j\right)\left(1 + \nu^j\right) + \lambda\right] f - \mu^j u_2'\left(w + m_2^j - y_2^j\right) f_{e^j} = 0, \tag{25}$$

$$\left(1 + \nu^j\right)\left[\int_{x^j}\int_{m_2^j} u_2 g_{d^j} dm_2^j f dx^j - z_{d^j}^j\right] + \lambda \int_{x^j}\int_{m_2^j} y_2^j g_{d^j} dm_2^j f dx^j - \mu^j z_{e^j d^j}^j = 0 \tag{26a}$$

[23] In Appendix A0, we show that W is concave. For brevity, we shall omit the arguments of v, u_1 and u_2, respectively $\left(k^j - m_1^j\right)$, $\left(m_1^j + y_1^j\right)$ and $\left(w_2 + m_2^j - y_2^j\right)$.

and

$$(1+\nu^j)\left\{\int_{x^j}\left[v+\rho\left(u_1+\int_{m_2^j}u_2 g dm_2^j\right)\right]f_{e^j}^j dx^j-\rho z_{e^j}^j\right\}$$
$$+\lambda\int_{x^j}\left[\int_{m_2^j}y_2^j g dm_2^j-y_1^j-p\right]f_{e^j}^j dx^j$$
$$+\mu^j\left\{\int_{x^j}\left[v+\rho\left(u_1+\int_{m_2^j}u_2 g dm_2^j\right)\right]f_{e^j e^j}dx^j-\rho z_{e^j e^j}^j\right\}=0, \quad (27)$$

where λ is the Lagrange-multiplier associated with (20), μ^j that associated with (22), and ν^j that associated with (23). The term $\int_{x^j}\int_{m_2^j}u_2 g_{d^j}dm_2^j f_{e^j}dx^j$ in (26a) is equal to zero because an increase in study effort, and consequently in the expected degree result, does not affect the marginal effect of d^j on date-2 expected utility (see Appendix A0, c).

Conditions (24) and (25) say that the principal chooses $\left(y_1^j, y_2^j\right)$ so as to equate the marginal benefit to the opportunity-cost of the payment to or from j at each date, and in each possible state of the world. The remaining conditions are slightly less straight-forward. The first LHS term of (26a) is the expected private benefit, and the second the expected external benefit (*via* the government budget constraint), of inducing j to take more science subjects. The third term is the marginal effect of d^j on the incentive-compatibility constraint. The first LHS term of (27) is equal to zero in view of the incentive-compatibility constraint (22). The second is the amount by which the principal's budget constraint is relaxed if e^j increases a little, and is the sum of two partial effects. One arises from the fact that, the harder j studies, the more he is likely to earn, and thus to pay the principal when he gets into the graduate labour market. The other arises from the fact that, the harder j studies, the more the principal is likely to have to subsidize him while a student. The third LHS term of (27) is proportional to the LHS of the second-order condition on j's choice of e^j, and thus negative.

6 First best

We define the first best as a hypothetical situation where individual study effort and parental support are observable by the principal, and there are thus no incentive-compatibility constraints. Let the FB subscript denote the first-best value of a variable. In Appendix A2, we show the following.

Proposition 2. *In first best,*

i) all school leavers with the right personal qualities become students;

ii) consumption is equalized across students, graduates, dates, and states of the "world,

$$m_1^j+y_1^j = w+m_2^j-y_2^j=c;$$

iii) at date 1*, the principal pays student* j *a sum,* $y_1^j = y_{1FB}\left(k^j\right)$*,* *where* $y_{1FB}(.)$ *is a decreasing function of parental income;*

iv) at date 2*, graduate* j *pays the principal a sum* $y_2^j = y_{2FB}\left(m_2^j\right)$*,* *where* $y_{2FB}(.)$ *is an increasing function of graduate earnings;*

v) parental consumption is equalized across parents and states of the world for parents who support their children,

$$k^j - m_1^j = C \text{ for } m_1^j = m(k^j, y_1^j) > 0,$$

equal to parental income, and lower than C*, for all others;*

vi) all type-i students choose the same subject mix, d_{FB}^i*, and supply the same effort,* e_{FB}^i*;*

vii) type-s students specialize completely in science, $d_{FB}^s = 1$*, and supply more effort than type-a* students, $e_{FB}^s > e_{FB}^a$*;*

viii) type-a students take fewer science subjects than type s, but do not specialize completely in the arts, $0 < d_{FB}^a < d_{FB}^s$ *.*

Therefore, in first best, all high-ability school leavers get to university. As the principal does not have to provide them with costly incentives to study hard, it could not in fact be optimal to leave any of them out. Where students are concerned, the first-best allocation is characterized by perfect equity, full insurance and perfect consumption smoothing. Having assumed descending altruism, this implies perfect equity also for those parents who contribute to their children's consumption at date 1. Parents not rich enough to subsidize their children consume all their income, and have lower expected utility than parents who do.[24] In standard principal-agent models, the full-insurance property descends from the assumption that the principal is less risk-averse than the agents. Here, by contrast, it is due to the fact that the principal does not face any budget uncertainty.[25] Since students differ only in their aptitude for science and arts subjects, all students of the same type behave the same. Although consumption is the same for both types, however, utility may not because the disutility-of-effort function is different.[26]

Notice that first-best policy redistributes not only from rich to poor students, but also from rich to poor graduates. It thus compensates students not only for any difference in the amount of support they receive from their parents, but also for any difference in the amount they will earn in the graduate labour market. That is indeed the reason why, in first best, the university participation constraint is satisfied for every agent. The payment that a student gets from the principal at date 1 depends only on his parent's income. Depending on parameter values, this payment could be positive for everyone, or positive for

[24] That is because we have defined the first best with respect to moral hazard only. To achieve a first best in the broadest sense, we would need personalized lump-sum taxes and subsidies.

[25] The same result emerges from Cigno *et al.* (2003)

[26] We cannot tell whether $z^s(e_{FB}^s, d_{FB}^s)$ is higher or lower than $z^a(e_{FB}^a, d_{FB}^a)$.

poor (low parental income), and negative for rich (high parental income) students. The payment that a graduate makes to the principal at date 2 depends only on his own earnings. Merit does not come into it because study effort is observable. As in *laissez faire*, both the study effort, and the science content of the degree, are higher for an s than for an a. That is because the expected monetary return to investing in a university education is higher for an s than for an a. Investing more in the education of scientifically inclined, than in that of artistically inclined students, will thus relax the principal's budget constraint. As a consequence of this investment, and of stochastic dominance, type-s graduates will earn, on average, more than type-a graduates. At date 2, the former will then pay the principal, on average, more than the latter (but a very successful arts graduate may well earn and pay more than a not-so-successful science graduate).

7 Second best

In the more realistic situation where individual student effort and parental support are not observable, the principal faces the incentive-compatibility constraints (21) − (22). Let the SB subscript denote the second-best value of a variable. In Appendix A3, we demonstrate the following.

Proposition 3. *In second best,*

i) not necessarily all school leavers with the right personal qualities become students;

ii) all students of the same type have the same expected consumption;

iii) at date 1*, the principal pays student* j *a sum,* $y^j_{1SB} = y^i_{11}\left(k^j\right) + y^i_{12}\left(x^j\right)$*, where* $y^i_{11}(.)$ *is a non-increasing function of parental income, and* $y^i_{12}(.)$ *an increasing function of degree result;*

iv) at date 2*, graduate* j *pays the principal a sum,* $y^j_{2SB} = y^i_{21}\left(m^j_2\right) + y^i_{22}\left(x^j\right)$ *, where* $y^i_{21}(.)$ *is an increasing function of graduate earnings, and* $y^i_{22}(.)$ *a decreasing function of degree result;*

v) all parents of students of the same type have the same expected consumption if they support their children through university, a lower one, equal to parental income, if they do not;

vi) all type-i students take the same subject mix, d^i_{SB}*, and supply the same amount of effort,* e^i_{SB}*;*

vii) type-s students specialize completely in science, $d^s_{SB} = 1$*;*

viii) type-a students take more arts subjects d^a_{SB} *than would be efficient conditional on* y^i_{tSB}*;*

ix) type-s students supply more effort than type-a students, $e^s_{SB} > e^a_{SB}$*, but both supply less effort than would be efficient conditional on* y^i_{tSB}*.*

Therefore, the second-best policy provides some, but not full insurance. Furthermore, it redistributes in favour of poor students, but equalizes expected consumption only within types. In other words, the policy fully compensates for differences in parental support only for students of the same type, because there would be no advantage in distorting the actions of students with the same personal qualities. Given descending altruism, this implies that parents of students of the same type will have the same expected consumption if they are rich enough to support their children through university, lower expected consumption if they are not. If they never support their children, their own consumption is simply equal to their own income.

The payment that a student receives from the principal at date 1 is the sum of two type-specific functions. The first is constant at parental income levels such that parental support is zero, decreasing in parental income at higher ones. As it depends solely on "need", this payment component redistributes and provides insurance. The second is increasing in academic performance ("merit"), and deals with moral hazard. The functions are designed to give the same marginal incentive to study hard to all students of the same type, but not to students of different types because they will have different disutility-of-effort functions. The payment that a graduate makes to the principal at date 2 is similarly the sum of two type-specific functions, the first increasing in graduate earnings to provide insurance, and the second decreasing in degree result. The rationale for offering a discount to graduates with better degree results is that the date-2 payment reduces net earnings and, therefore, the incentive to study hard at date 1. Since j does not know, at date 1, how much he will earn at date 2, the marginal disincentive effect is independent of the realization of m_2^j.

The second-best policy encourages students with a penchant for the arts to specialize in their favourite subjects more than would be efficient given the second-best payment schedules. The intuitive explanation is that, since effort is not observable, and inducing a student to study hard is thus costly, the principal must make it easier for the student to get better degree results. For type-a students, whose marginal disutility of effort is increasing in the science content of the degree, the incentive constraint is relaxed by reducing the science content. For type-s students, by contrast, it is relaxed by increasing the science content of the degree, and this is why they will take only science subjects as in first best. Therefore, the second-best policy does not distort the subject mix chosen by students predisposed to the study of science. Regarding the choice of effort level, we can say that this will always be inefficiently low, because the agents do not take into account the social benefit (i.e., the effect on the principal's budget constraint) of individual effort. Notice further that, given d_{SB}^i, effort is less costly for the scientific type, who is taking his least-cost subject mix, than for the artistic one, who is not. That is why the second-best choice of effort is higher for the former than for the latter.

Can we be sure that, at date 2, type a will pay on average less than type s as in first best? The answer is no. Since x^a and x^s have the same distribution (in other words, greater effort has the same effect on the probability of getting a good degree result for both types), and having established that e_{SB}^a is lower than

e^s_{SB}, the expected x^a_{SB} is in fact lower than the expected x^s_{SB}. This *may* modify the first-best conclusion that type-s graduates must pay more, on average, than type-a (indeed, as we have seen, y^j_{2SB} is increasing in m^j_2, but decreasing in x^j). The possibility that the average type-a graduate will pay more than the average type-s graduate cannot thus be ruled out.

Let us now consider the possibility that the university-participation constraint (23) is binding for some agents. We know that this could not happen in first best. It could happen in second best, however, because the cost of providing a student with the incentive to supply the desired level of effort reduces the expected gain from a university education. Note that this is the same for all students of the same type irrespective of parental income. By contrast, the utility of going straight into the labour market is increasing in k. We cannot thus rule out the possibility that relatively rich students of either type will be participation-constrained. This has an important implication. A participation-constrained student, h, enjoys the same level of expected utility irrespective of whether he goes to university or straight into the labour market (and the same is true of the student's parent). If h is expected to be a net contributor,[27] he should be kept in the public scheme (i.e., offered the y^h_1 which satisfies the participation constraint as an equation).[28] Otherwise, social welfare will be maximized by letting h go straight into the labour market (i.e. , offering h a y^h_1 that violates the participation constraint), and using the resources thus freed to raise the expected utility of unconstrained agents. We cannot even rule out the possibility that all agents of the same type are participation-constrained (in which case only one type of student would get public support, and a high-ability school leaver of the other type would go to university if and only if his parent were rich enough to make him a net contributor to the scheme), but this seems highly unlikely. Since the cost of providing a student with the right incentive is lower, and the amount he is expected to earn in the graduate labour market higher, for an s than for an a, if a type is going to be denied support this will be the artistic one.

Let us now relax the assumption that the random variables are i.i.d.. If the degree results of different agents are "affiliated and dependent" in the sense of Milgrom and Weber (1982),[29] both the date-1 and the date-2 payments due to or from j will be optimally functions of the vector $x = \left(x^j, x^{-j}\right)$ because the degree results of students other than j convey information about j's own study effort.[30]

[27] A student j is a net contributor if $\int_{x^j} \left(\int_{m^j_2} y^j_{2SB} g^j dm^j_2 - y^j_{1SB} - p \right) f^j dx^j > 0$. Recall that y^j_1 can be negative for rich students, implying that such students are offered a gross transfer, $\chi^j \geq 0$, not large enough to cover the tuition fee.

[28] Strictly speaking, therefore, properties ii) to ix) of the second-best policy outlined in Proposition 3 apply only to participation-unconstrained students.

[29] Affiliation implies that the random variables tend to "move together". In other words, it is more likely that the realized values will be all high, or all low, than that some will be high, and others low. Since affiliated random variables have nonnegative covariance, affiliation includes, as a special case, independence.

[30] Consider the conditions determining the form of $y^i_1(.)$ and $y^i_2(.)$, (40) and (41), in the proof of Proposition 3. If the random variable affecting j's degree result is stochastically dependent on the random variables that affect the degree results of other students, the likelihood ratio

It can be shown that y^j_{1SB} is increasing in x^j, and decreasing in each element of x^{-j},[31] and that y^j_{2SB} is decreasing in x^j, increasing in each element of x^{-j}, and independent of m_2^{-j}. The implication is that the policy maker should look only at relative degree results, and not be fooled into making higher net payments to all students, or requiring lower payments from all graduates, if all degree results drift upwards. The intuition is that, if degree results move together, this may reflect the behaviour of universities or individual examiners (for example, competition for student numbers could lead to a general lowering of examination standards), rather than the behaviour of the students. By contrast, the policy maker should look only at a graduate's own earnings because, for any given degree type and degree result, these will depend only on chance.[32]

8 Policy interpretation: scholarships or loans?

If we interpret

$$\chi^j \equiv y_1^j + \theta^j \tag{28}$$

as a scholarship, and y_2^j as a graduate tax, the following is obviously true.

> **Proposition 4**. *A first or second best can always be implemented using a scholarship scheme financed by a graduate tax.*

Alternatively, interpret χ_1^j as a loan, and y_2^j as a loan repayment. Compared with scholarships and graduate taxes, loans face the policy maker with additional restrictions. One is a necessary feature of a credit scheme and says that, since j can escape having to repay a loan by not accepting one in the first place, the policy must satisfy the credit-participation constraint

$$\pi^j_{S/PS}\left(k^j, y_1^j, y_2^j\right) \geq \pi^j_{S/LF}\left(k^j, -\theta^j\right), \tag{29}$$

where the PS subscript denotes the value of a variable if the student is included in the public loan scheme. No such a restriction applies to a scholarship scheme because j cannot escape paying the graduate tax.[33] By itself, (29) would not be an obstacle to the achievement of either a first or second best, as the case may be, because the policy maker could always fix y_1^j at the optimal level, and then raise χ^j and θ^j simultaneously until (29) is satisfied. But this may not be possible if there are further restrictions.

$f_{e^j}(x|e)/f(x!e)$ is a function of the entire $x = \left(x^j, x^{-j}\right)$ vector (Holmström, 1982).

[31] That y^j_{1SB} is increasing in x^j when $\frac{f_{e^j}(x,!e)}{f(x!e)}$ is monotone in x^j is a well known result; that it is monotonically decreasing in x^{-j} if and only if the random variables are affiliated is demonstrated in Luporini (2006).

[32] That would not remain true if we allowed the probability distribution of graduate earnings to be conditional not only on degree type and degree result, but also on search intensity or work effort.

[33] As the principal can always use a forcing contract to prevent j from choosing d^j_{LF} instead of d^j_{FB} or d^j_{SB}, it will never be in j's interest to refuse a scholarship.

One such restriction could be that universities are not allowed to charge a student more than the average cost,

$$\theta^j(d^j) \leq p. \tag{30}$$

We may interpret this as a political ("no-implicit-education-tax") constraint, which allows the policy maker to offer discounts to poor students, but not to overcharge rich ones. Such a constraint could be in place even if χ^j were a scholarship, and y_2^j a graduate tax. In that case, however, it would not be an obstacle to the achievement of a social optimum because all that would matter then is the difference between χ^j and θ^j, not their absolute values. If χ^j is a loan, by contrast, (30) will make (29) binding for students from richer families, and thus prevent the achievement of the appropriate (first or second best) social optimum. In Appendix A4 we prove the following.

> **Proposition 5**. *A loan scheme is at best as good as a scholarship scheme financed by a graduate tax.*

Another political ("no-usury") constraint could be that the policy maker must not charge students more than the market rate of interest. Using (28), and recalling that date-2 money variables are discounted back to date 1 at the market rate of interest, we can write this constraint as

$$y_2^j \leq \chi^j \ . \tag{31}$$

In the absence of (30), (31) would make no difference to the results because the policy maker could again fix y_1^j at the optimal level, and then raise χ_1^j and θ^j simultaneously until both (29) and (31) are satisfied. We show at the end of Appendix A4 that, in the presence of (30), (31) will be binding, and will thus make welfare even lower (deny a higher education to an even greater number of high-ability school leavers) than if the policy maker were free to charge richer students more than the market rate of interest. Note that (31) is not a necessary feature of, but can arise only in connection with, a loan scheme.

The justification for hypothesizing (30) and (31) is that student loans derive their political popularity from the fact that, unlike scholarships, they do not involve extra taxation. If the latter are rejected in favour of the former, it then seems logical that the legislator will forbid the government to use fees and repayment terms as if they were taxes. Is there anything we can say about what kind of student would be excluded from a loan scheme?

Ignore, for the moment, the no-usury constraint. Recall that, if the no-implicit-education-tax constraint is binding for some agent, the credit-participation constraint will be binding too. If $(29) - (30)$ are binding for anyone, it will be for students from rich families who can finance their studies entirely out of parental resources (i.e., given type i, for $k^j \geq \widetilde{k}_{LF}^i$). The expected utility of these students is the same whether or not they get a loan. The policy maker wants them in the credit scheme, however, because he expects them to be net contributors.[34] Since

[34] For the definition of net contributor *s*ee note 27 above. Since students taking a loan will gain from consumption smoothing, net contributors are not necessarily credit constrained

the net contribution will be lower than it would be without (29), however, there will be less to redistribute than if the policy were a scholarship-*cum*-graduate-tax scheme. If effort were observable, that would have no implications for the number of agents attending university, because all high-ability school leavers would gain from consumption-smoothing, and from insurance. As it is not observable, however, part of the policy maker's budget will have to be used for providing costly incentives. Some school leavers from poor families may thus be denied credit, and consequently excluded from university, but their number will be lower than in *laissez faire* because the policy will still do some redistribution, and provide some insurance.

The policy we have just outlined is not feasible in the presence of the no-usury constraints, (31), because it would imply charging students from rich families higher tuition fees than the rest. If these fees cannot be higher than p, there will be something to redistribute only if the scheme is subsidized by the general tax payer. The same may be said of the scheme's ability to provide insurance. Therefore, the policy needs S to cover the cost of setting

$$y_2^j < y_1^j + p$$

for some j, in some state of nature. Without it, poor students could not be charged less than average total cost, and unlucky graduates could not be allowed to pay back less than the capitalized value of the money borrowed. If the higher education externality and, consequently, S are not sufficiently large, some school leavers will not be offered a loan. Which ones? Recall that (30) has the effect of making (29) binding. Recall, also, that the principal wants students from rich families in the credit scheme only if they are expected to be net contributors, but the constraint on the size of the loan repayment prevents that. Furthermore, since loan repayments are income contingent, all students, including those with rich parents, must be subsidized in unfavourable states of nature. Therefore, the richest school leavers will be excluded from the scheme. If S is sufficiently low, the poorest ones will be excluded too because the policy maker will not have enough money to subsidize them. Unlike the richest, however, the poorest will be excluded not only from the credit scheme, but also from university. Since type-a agents are expected to have lower graduate earnings, and thus to make smaller repayments, the poor school-leavers excluded are likely to be those with a penchant for the arts.

If S is zero,[35] loan repayments cannot be income contingent. We can then have only mortgage-type loans, without any redistributive or insurance element. Such a scheme will exclude any agent who would not be able to pay back the full capitalized value of his loan in the worst possible state of the world. Since it will allow at least some agents to discount their expected graduate earnings, however, it will still be better than *laissez faire.*

even in the presence of (30). For the same reason, however, credit-constrained agents are net contributors.

[35] In real life, all student loan schemes are subidized out of general tax revenue

9 Discussion

We started from the premise that, in the absence of full contingent markets, a number of school leavers who would otherwise choose to invest in a university education will not do so. There is thus scope for raising social welfare by allowing young persons with the right personal qualities to discount the expected return from a university education, and to insure against academic and graduate labour market risks. The question is how. The language used in the public debate over scholarships and student loans seems deliberately designed to obscure the difference between the two. For example, the expression ”income-contingent loan” is used to cover a variety of situations, ranging from conventional mortgage-type loans combined with some earnings insurance, to schemes where the so-called loan repayment bears little or no relation to the size of the so-called loan. In interpreting the analytical solution of the social optimization problem, we have adopted the convention of calling the policy a credit scheme if a person can avoid incurring any extra charge after graduating by simply turning down any public offer of financial support while a student. Otherwise, we call the public support received by a student a scholarship, and the extra charge incurred by a graduate for the very fact being a graduate a tax. Irrespective of whether it is interpreted as a scholarship or a loan, the second-best payment due to a student of the right ability has a "need " and a "merit" component. The former is *)*non-increasing in parental income (decreasing if the parent is rich enough to support him through university), and could be paid out at front. The latter is increasing in degree result, and would have to be paid in installments as exam results come in. This contrasts with the finding in Hanushek et al. (2004) that need-based grants perform better than merit-based ones. The difference is due to the fact those authors do not consider moral hazard. In our framework too, if individual effort were observable (i.e., in first best), the optimal grant would depend only on need. We also find that the second-best payment due from a graduate has two components, one increasing in graduate earnings, and the other decreasing in degree result. Both payments take account of whether the individual has a comparative advantage in the study of "science", or in that of "the arts" (short-hand for high or low expected earnings).

Since the policy maker maximizes a sum of expected utilities, rather than total tax revenue, it is not socially desirable that everyone should specialize in the study of science. Those with a scientific bent should specialize completely in the study of science. Those with an artistic one should specialize in the study of the arts, but not completely because there is a trade-off between personal cost (disutility of study effort) and expected graduate earnings. For the same reason, in first and second best as in *laissez faire*, artistically-inclined students study less hard than scientifically-inclined ones. In second best, the former will also take fewer science subjects than would be efficient conditional on the policy because that will relax the policy maker's incentive-compatibility constraints. However, the second-best policy may exclude some artistic school leavers from poor families. Those from rich families will subsidize poor students of the scientific type. If the random components of the degree results of the different students tend to move in the same direction, the second-best payment due to a student, and the second-best payment due from a graduate, must take account not only of the person's own degree result, but also of those obtained by others. The implication is that the policy maker should not be fooled by grade drift into granting every student a higher scholarship or loan, and charging every graduate a lower tax or loan repayment. By contrast, the second-best tax or loan repayment due from a graduate will optimally depend only on this person's own earnings.

The straight answer to the title question is that a scholarship scheme financed by a tax on graduate earnings (or a graduate surcharge on general income tax) is always at least as good as a student loan scheme. Indeed, the former will implement the first best allocation if individual effort and parental support are observable by the policy maker, the second best one if they are not. Garcia-Peñalosa and Wälde (2000) argue that an income-contingent loan scheme subsidized by general taxation is sub-optimal because it is regressive; we have shown that it may be sub-optimal even if it is not regressive. The reason why a loan scheme may not achieve as good a result as a scholarship scheme is that the terms of the credit contract must be such, that it will be in the student's interest to accept it. This constraint, absent in the case of a scholarship scheme, is not by itself sufficient to make loan schemes inferior. For that to be the case, there must be another restriction, namely that tuition fees cannot be higher than average total cost (in other words, that universities may be allowed to practice discounts to deserving students, but not to overcharge the rest). This additional restriction is likely to be in place if a scholarship scheme is politically unacceptable because it would involve extra taxation, and effectively says that the policy maker will not be allowed to use tuition fees as if they were taxes. In the presence of such a restriction, some poor school leavers with the right intellectual qualities may be denied a loan, and thus a higher education. If, in addition to the ceiling on tuition fees, there is also the restriction that a student cannot be charged more than the market rate of interest, this will have the effect of excluding not only some poor, but also some rich agents from the scheme. Unlike the poor, however, the rich will not be excluded from university. The justification for this further restriction is again that, if the policy maker is not allowed to tax graduates openly, he will not be allowed to tax them surreptitiously by charging them excess interests.

The greater is the higher education externality, and thus the extent to which the policy is subsidized by the general tax payer, the higher is the expected welfare gain associated with the policy. That is true in general, but more so if the policy is a loan scheme, and even more if the policy maker is not allowed to use differential tuition fees or loan repayment terms to cross-subsidize poor students or unlucky graduates (for, in that case, redistribution and insurance is possible only to the extent that the general tax payer is willing to pay for it). Without such a subsidy, loans could not be income-contingent, and would then have to be paid back in full, at market interest rates, as in a conventional mortgage. The outcome would still be better than *laissez faire* because the policy would allow at least some talented young people from poor families, who otherwise would not go to university, to get a higher education. In practice, however, all student loans are income-contingent, because there is no way of extracting a repayment from either a failed student, or an unlucky graduate worker. Therefore, it is actually impossible for a non-usurarious loan scheme to be entirely self-financing.

These results were obtained under the simplifying assumption that parents can observe their children's study effort. This is what allowed us to model the policy optimization as an agency problem with the policy maker in the role of principal, and the parents of high-ability school leavers in that of agents. The assumption is more easily justifiable if students live at home, but retains some validity even if they do not, because parents still have an informational advantage over any public authority. What happens if we abandon the assumption that parents observe their children's study effort? If we do, the student is the effective agent. Recall that the second-best parental support depends on child resources, and that parental utility is consequently increasing in the student's degree result. It is thus in the parent's interest to use the level of parental support as a means for inducing the student to study harder. This relaxes the policy maker's incentive compatibility constraint, but the qualitative results are not otherwise affected.

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Appendix

A0. Proof of concavity of the expected utility functions, and of the social welfare function

The expected utility function of each parent is concave in m_1, y_1, and convex in y_2, for the assumption that $v(.)$ and $u_t(.)$ are concave. To show that the expected utility function is concave also in d, and in e, we write a student's expected time-2 utility conditional on x as

$$\omega(x,d) \equiv \int_{m_2} u_2(w + m_2 - y_2)\, g dm_2, \tag{32}$$

with $y_2 \equiv 0$ in *laissez faire*. The function $\omega(.)$ has the following properties:

a) $\omega_x(x,d) > 0$, because $G_x(m_2|x,d) \leq 0$, with the strict inequality sign holding for some value of m_2 at each x (i.e., because of FOSD of $G(m_2|x,d)$ for higher values of x), given that $u_2()$ is increasing in m_2;

b) $\omega_d(x,d) > 0$, again because $G_d(m_2|x,d) \leq 0$, with the strict inequality sign holding for some value of m_2 at each d (i.e., because of FOSD of $G(m_2|x,d)$ for higher values of d), given that $u_2()$ is increasing in m_2;

c) $\omega_{dx}(x,d) = 0$, because $G_{dx} = 0$ implies that an increase in x does not affect the FOSD effect of an increase in d with regard to the distribution of m_2. Note that this also implies that $\int_x \omega_d(x,d) f_e dx = \int_x \int_{m_2} u_2 g_d dm_2 f_e dx = 0$.

The parent's expected utility function can be written as

$$\int_x [v(k - m_1) + \rho(u_1(m_1 + y_1) + \omega(x,d))]\, f dx - \rho z^i(e,d), \tag{33}$$

with $y_1 = -p$ and $m_1 = m(k,-p)$ in *laissez faire*. To show that this function is concave in e, let us integrate the above expression by parts. Recalling that $x \in X = [\underline{x}, \overline{x}]$ and using (14) and (16), we obtain

$$\begin{aligned} & v(k - m(k, y_1(\overline{x}))) \\ & + \rho\left\{u_1\left[m(k, y_1(\overline{x})) + y_1(\overline{x})\right] + \omega(\overline{x}, d)\right\} \\ & - \rho\left[\int_x \left(u_1'(m_1 + y_1)\frac{\partial y_1}{\partial x} + \omega_x(x,d)\right) F(x|e)dx + z^i(e,d)\right] \end{aligned}$$

The first two terms are constant. The term in brackets under the integral sign is positive, because $u_1' > 0$, $\partial y_1/\partial x > 0$ (= 0 in laissez faire) from the MLR assumption (see the proof of proposition 3 below), and $\omega_x(x,d) > 0$. Considering that $F(x|e)$ is convex in e for the CDF assumption, the expression under the integral sign is also convex in e. Since $z^i(e,d)$ is also convex in e, it follows that the expected utility function is concave in e.

Similarly, to prove that expected utility is concave in d, let us integrate the RHS of (32) by parts with respect to m_2. Recalling that $m_2 \in M_2 = [0, \overline{m_2}]$, we obtain

$$\omega(x,d) \equiv u_2(w, \overline{m_2}) - \int_{m_2} u_2'(w + m_2)\, G(m_2|e,d)dm_2.$$

The first term is again constant, and the expression under the integral sign is convex in d, because $u_2^{'}$ is constant with respect to d, and $G(.|e,d)$ is convex because of the CDF assumption. Since $z^i(e,.)$ is convex too, it then follows that the expected utility function is concave in d. As the expected utility function of each parent is concave, the social welfare function, which is the sum of individual utility functions, will be concave too.□

A1. Proof of Proposition 1

We first prove parts (ii) and (iii), and then use the results to prove part (i).

A1.1. Proof of parts (ii) and (iii)

At an interior solution, the first-order conditions for (18) w.r.t. (d,e) are

$$z_d^i(e,d) = \int_x \int_{m_2} u_2(w+m_2)\, g_d dm_2 f dx \tag{34}$$

and

$$z_e^i(e,d) = \int_x \int_{m_2} u_2(w+m_2)\, g dm_2 f_e dx \equiv \int_x \omega(x,d)\, f_e dx. \tag{35}$$

In view of (34) and (35), (d_{LF}, e_{LF}) depends only on the expected date-2 gain. Since this expectation is the same for all students of the same type, all type-i students will choose the same subject mix and effort level, $\left(d_{LF}^i, e_{LF}^i\right)$. Where type a is concerned, the choice of subject mix is always interior, because $z_{d^a}^a > 0$. Therefore, $0 < d_{LF}^a < 1$. Where type s is concerned, the choice of subject mix is always at a corner, because $z_{d^s}^s < 0$. Therefore, $d_{LF}^s = 1$.

The effort level e_{LF}^i is determined by (35) for $i = a, s$. To prove that $e_{LF}^s > e_{LF}^a$, suppose that the s type chooses $e^s = e_{LF}^a$. Given the assumptions 2, 3 and 6 on the form of $z^i(e,d)$, this implies that the LHS of equation (35) is lower for the s than for the a type. On the other hand, the RHS is higher for the s than for the a type, because of b) and c) in **A0**. Since the a type is choosing optimally, we know that (35) holds for $i = a$. As a consequence, for the s type, the RHS would be higher than the LHS, implying that the e^s must be raised above e_{LF}^a for (35) to hold. As e^s rises, z_e^s will in fact increase because of assumption 3, while $\int_x \int_{m_2} u_2(w+m_2)\, g dm_2 f_e dx$ will decrease because $\int_x \int_{m_2} u_2(w+m_2)\, g dm_2 f dx$ is concave in e.□

A1.2. Proof of part (i)

A high-ability school leaver will not go to university if either $m(k,-p) < p$, or $\pi_{S/LF}^i(k,-p) < \pi_W^i(k)$. Clearly it cannot be $m(k,-p) > p$, for $k < p$. Given that $v'(0) = u_1'(0) = \infty$ and $1 > \rho > 0$, (14) implies $m(k,-p) = 0$ for $k = p$. However, there will be a value $\widehat{k} = p + \varepsilon$ in a neighborhood of p, such that (14) implies $m(k,-p) > p$ for $k \geq \widehat{k}$.

Note that, for any $k \geq \widehat{k}$, $m_I < 0$ implies that $\frac{\partial \pi_{S/LF}^i(k,-p)}{\partial k} = v'(k - m(k,-p)) > \frac{\partial \pi_W(k)}{\partial k} = v'(k - m(k,w))$. There are then two possible cases.

(i) if $\pi_{S/LF}^i(\widehat{k},-p) \geq \pi_W(\widehat{k})$, then $\widetilde{k}_{LF}^i = \widehat{k}$;

(ii) if $\pi_{S/LF}^i(\widehat{k},-p) < \pi_W^i(\widehat{k})$, raising k above $\widehat{k}$ will bring us to a level, $\widetilde{k}^i$, such that $\pi_{S/LF}^i(\widetilde{k}^i,-p) = \pi_W(\widetilde{k}^i)$; then $\widetilde{k}_{LF}^i = \widetilde{k}^i$.

Now consider (18) and note that, for any given k, the date-1 utility from income is the same for both types. However, it must be the case that

$$\int_x \int_{m_2} u_2\left(w+m_2\right) g(m_2|e^s_{LF},1)dm_2 f(x|e^s_{LF})dx - z^s(e^s_{LF},1) >$$
$$\int_x \int_{m_2} u_2\left(w+m_2\right) g(m_2|e^a_{LF},d^a_{LF})dm_2 f(x|e^a_{LF})dx - z^a(e^a_{LF},d^a_{LF}),$$

otherwise a type-s student could increase his utility by choosing $e^s = e^a_{LF}$, and e^s_{LF} would not be optimal. It then follows that $\widetilde{k}^a > \widetilde{k}^s$. Consequently, either $\widetilde{k}^i_{LF}=\widehat{k}$, $i = a, s$, or $\widehat{k} \leq \widetilde{k}^a_{LF} < \widetilde{k}^s_{LF}$.□

A2. Proof of Proposition 2

A2.1. Proof of part (i)

By definition, a high-ability school leaver is one who would go to university, without public or parental help, if he were able to trade in a complete system of contingent markets. In view of postulate (x), and of the fact that the principal does not face any budget risk, it is obvious that the first-best policy will allow all these talented young persons to attend university.

A2.2. Proof of parts (ii), (iii), (iv) and (v)

In first best, $\mu^j = 0$ for all j. Therefore, given $\nu^j = 0$, (24) reduces to

$$u_1'\left(m(k^j, y_1^j) + y_1^j\right) = \lambda, \tag{36}$$

and (25) to

$$u_2'\left(w + m_2^j - y_2^j\right) = \lambda, \tag{37}$$

implying that consumption is the same at both dates and for all j, irrespective of k^j, and of the realizations of x^j and m_2^j,

$$m(k^j, y_1^j) + y_1^j = \ w + m_2^j - y_2^j = c.$$

On the other hand, in view of $-1 < m_I < 0$, (36), where $I = y_1^j$, implies part (iii) of the proposition. Analogously (37) implies part (iv). Part (v) then follows from the parent's first-order condition (14).

A2.2. Proof of parts (vi), (vii) and (viii)

Using $\mu^j = \nu^j = 0$, (36), (37), (14) and

$$\int_{m_2^j} g^j_{d^j} dm_2^j = \int_{x^j} f^j_{e^j} dx^j = 0,$$

the conditions on d^j and e^j, (26a) and (27), may be written as

$$z^j_{d^j} = \lambda \int_{x^j} \int_{m_2^j} y_2\left(m_2^j\right) g^j_{d^j} dm_2^j f^j dx^j \tag{38}$$

and

$$z_{e^j}^j = \lambda \int_{x^j} \int_{m_2^j} y_2 \left(m_2^j\right) g dm_2 f_{e^j} dx^j \tag{39}$$

In view of (38) and (39), at an interior solution, $\left(d_{FB}^j, e_{FB}^j\right)$ depend only on the expected date-2 welfare gain. Since this expectation is the same for all the students of the same type, all type-i students will choose the same subject mix and effort level, $\left(d_{FB}^i, e_{FB}^i\right)$. Where type a is concerned, the choice of subject mix is always interior, because $z_{d^a}^a > 0$. Therefore, $0 < d_{FB}^a < 1$. Where type s is concerned, the choice of subject mix is always at a corner, because $z_{d^s}^s < 0$. Therefore, $d_{FB}^s = 1$.

The level of effort e_{FB}^i is determined by (39) for $i = a, s$. Note that $y_2(.)$ is increasing in m_2^j like $u_2(.)$, and $\int_{x^j} \int_{m_2^j} y_2 \left(m_2^j\right) g dm_2^j f dx^j$ concave in e^j like $\int_{x^j} \int_{m_2^j} u_2 \left(w + m_2^j\right) g dm_2^j f dx^j$. As a consequence we can follow the same line of reasoning as in the proof of b) and c) of A0, to show that b') $\int_{m_2^j} y_2 \left(m_2^j\right) g_{d^j}^j dm_2^j > 0$ and c') $\int_{x^j} \int_{m_2^j} y_2 \left(m_2^j\right) g_{d^j} dm_2 f_{e^j} dx^j = 0$. To prove that $e_{FB}^s > e_{FB}^a$, suppose that the s type chooses $e^s = e_{FB}^a$. Given assumptions 2, 3 and 6 on z^i, this implies that the LHS of (39) is lower for the s than for the a type. On the other hand, given b') and c'), the RHS is higher for the s than for the a type. Since type-a students are choosing optimally, we know that (39) holds for $i = a$. As a consequence, for the s type, the RHS would be higher than the LHS, implying that e^s must be raised above e_{FB}^a for (39) to hold. As e^s rises, $z_{e^j}^s$ increases because of assumption 3, while $\int_{x^j} \int_{m_2^j} y_2 \left(m_2^j\right) g dm_2^j f_{e^j} dx^j$ decreases for concavity of $\int_{x^j} \int_{m_2^j} y_2 \left(m_2^j\right) g dm_2^j f dx^j$.□

A3. Proof of Proposition 3

We prove first parts (ii)-(ix), and then use the results to prove part (i).

A3.1. Proof of parts (ii)-(v)

To check that μ^j is positive, we re-write the first-order condition on y_1^j, (24), as

$$\frac{\lambda}{u_1'\left(m(k^j, y_1^j) + y_1^j\right)} = 1 + \nu^j + \mu^j \frac{f_{e^j}(x^j|e^j)}{f(x^j|e^j)}, \tag{40}$$

where ν^j is equal to zero if the participation constraint (15) is not binding. Given that $\frac{f_{ej}}{f^j}$ is increasing in x^j for the MLR assumption, if $\mu^j \leq 0$, the agent would always choose the lowest possible level of effort. With $\mu^j < 0$ $(= 0)$, the RHS of (40) is decreasing (constant) in x^j, while, the LHS is increasing in $m(k^j, y_1^j) + y_1^j$, itself an increasing function of y_1^j because $-1 < m_I < 0$. Hence, y_1^j should be a decreasing function of x^j, but this implies that the agent will always choose the lowest possible level of effort. Hence, μ^j is positive.

Given $\mu^j > 0$, y_1^j depends not only on x^j, but also on k^j. Consider that f_{e^j}/f and, therefore, the RHS of (40), is increasing in x^j, while the LHS is increasing in $m(k^j, y_1^j) + y_1^j$. Given that $-1 < m_I < 0$, y_1^j is then increasing in

x^j. Given that $m_k > 0$ if $m(k^j, y_1^j) > 0$, y_1^j decreasing in k^j for parental income levels such that $m(k^j, y_1^j) > 0$ (constant for lower levels of parental income).

The first-order condition on y_2^j , (25), may similarly be re-written as

$$\frac{\lambda}{u_2'\left(w + m_2^j \ - y_2^j\ \right)} = 1 + \nu^j + \mu^j \frac{f_{e^j}(x^j|e^j)}{f(x^j|e^j)}. \tag{41}$$

Since the LHS of (41) is increasing in the argument of u_2', y_2^j is again increasing in m_2^j. Since the RHS of (41) is increasing in x^j, and u_2' in y_2^j, the latter is decreasing in x^j.

The additively-separable form of the date-1 payment function derives from the fact that k^j does not enter the RHS of (40), and that m_k and m_I are constant. The additively-separable form of the date-2 payment function derives from the fact that m_2^j does not enter the RHS of (41). As a consequence, the argument of $u_t'(.)$ varies with x^j, but not with k^j and m_2^j. Since different types of student have different disutility-of-effort functions, $\mu^a \neq \mu^s$, implying that there is a different $y_t^i(.)$ for each i.

Part (v) follows again from the parent's first-order condition, (14).

A3.2. Proof of parts (vi)-(ix)

Given that agents are compensated for differences in k and m_2, all type-i students make the same choice of d^i and e^i. Given $z_{d^s}^s < 0$, it then follows that $d_{SB}^s = 1$. The level of d_{SB}^a is determined by ($26a$). Given that the last LHS term of ($26a$), $\mu^j z_{e^j d^j}^j$, is positive, d_{SB}^a is lower than would be optimal given the $y_t^a(.)$ schedules.

Given $d_{SB}^s = 1$ and $0 < d_{SB}^a < 1$, $z^i(e, d_{SB}^i)$ is lower for $i = s$ than $i = a$. Moreover, e^s has a higher positive effect on the student's expected income, and thus on the principal's budget constraint, than e^a. It then follows that $e_{SB}^s > e_{SB}^a$, where e_{SB}^i is determined by the incentive-compatibility constraint (22). That e_{SB}^i is inefficiently low given the $y_t^i(.)$ schedules, follows from the fact that (22) does not include the marginal effect of e_{SB}^i on the principal's budget constraint, measured by the second LHS term of (27). Such effect is positive because $\lambda > 0$, the first LHS term of (27) is equal to zero, and the last one is negative.□

A3.3. Proof of part (i)

We know from A3.1 that $\pi_S^i\left(k, y_{1SB}^i, y_{2SB}^i\right) \equiv \overline{\pi}_{SSB}^i$ is the same for all parents who support their children through university in all states of nature, and increasing in k (but lower than $\overline{\pi}_{SSB}^i$) for those who are not. We also know that the second-best expected gain from becoming a student, $H^i \equiv \left[\left(u_1\left(m(k, y_{1SB}^i) + y_{1SB}^i\right) - z^i\left(e_{SB}^i, d_{SB}^i\right) + \int_{m_2} u_2\left(w + m_2 - y_{2SB}^i\right) g dm_2\right)\right]$, is the same for all type-i students. On the other hand, we know from A1.2 that $\pi_W(k) = v(k - m(k, w)) + \rho[u_1(w + m(k, w)) + u_2(w)]$ is increasing in k. Since providing students with the incentive to study hard is costly, we cannot rule out that y_{1SB}^i is so low, or y_{2SB}^i so high, that H^i is sufficiently low to make (23) binding for type-i agents with relatively high k. If it so happens that H^i=$[u_1(w) + u_2(w)]$, all type-i agents, even those with $k = 0$, will be participation-constrained. Since it cannot be optimal for the policy maker to make everyone indifferent between going and not going to university (because the policy maker has always the option of doing nothing, in which case some at least would be better-off going to university), H^i=$[u_1(w) + u_2(w)]$ can only be true for either $i = a$ or $i = s$, not both. Since the distribution of x is the same for both types, but $z^i(e, d^i_{SB})$ is lower, and the expected m^i_2 higher, for $i = s$ than $i = a$, the only possibility is H^a=$[u_1(w) + u_2(w)]$. Let h be a participation-constrained agent. Since he enjoys the same level of expected utility irrespective of whether he goes to university or straight into the labour market, h will be kept in the scheme (i. e., offered the y^h_{tSB} which satisfies (23) as an equation) if and only if he is expected to be a net contributor, $\int_{x^h} \left(\int_{m^h_2} y^h_{2SB} g^h dm^h_2 - y^h_{1SB} - p \right) f^h dx^h > 0$. Otherwise, social welfare could be raised by setting y^h_t so low, that h would go straight into the labour market, and using the resources thus freed to raise the expected utility of unconstrained school-leavers.□

A4. Proof of Proposition 5

A first-best loan scheme offers j the loan schedule $\chi^j = \theta^j + y_{1FB}(.)$, and loan-repayment schedule $y_{2FB}(.)$. The tuition fee, θ^j, (and, consequently, the loan, χ^j) is undetermined.

Without (30) and (29), the principal can use θ^j to satisfy the credit participation constraint, (29). Take a type-i agent, j, for whom $\pi^j_{S/PS}\left(k^j, y^j_{1FB}, y^j_{2FB}\right) < \pi^j_{S/LF}\left(k^j, p\right)$. This is an agent who, for $\theta^j = p$, would prefer not to participate in the loan scheme. The principal wants j in because he would be a net contributor, $\int_{x^j} \left(\int_{m^j_2} y^j_{2FB} g^j dm^j_2 - y^j_{1FB} - p \right) f^j dx^j > 0$. To keep him in, however, the principal would have to charge j a personalized tuition fee contingent on the subject mix, $\theta^j(.)$, such that (a) $\theta^j\left(d^i_{FB}\right)$ is sufficiently high for it to be true that $\pi^j_{S/PS}\left(k^j, y^j_{1FB}, y^j_{2FB}\right) > \pi^j_{S/LF}\left(k^j, \theta^j(d^i_{FB})\right)$, and (b) $\theta^j(d^j)$ is sufficiently higher than $\theta^j(d^i_{FB})$ for it to be true that $\pi^j_{S/PS}\left(k^j, y^j_{1FB}, y^j_{2FB}\right) > \pi^j_{S/LF}\left(k^j, \theta^j(d^j)\right)$ for any $d^j \neq d^i_{FB}$. That will not be possible, however, if (30) prevents the principal from setting tuition fees higher than p. If (30) is binding, (29) will be binding too. Similar arguments apply to the second best. All we have to do is replace y_{tFB} and d^i_{FB} with y_{tSB} and d^i_{SB}.

Therefore, a loan scheme faces a loan-participation constraint, (29), which is not present in a scholarship scheme. This additional constraint will be binding if there is also a no-implicit-education-tax constraint, (30). In this case, the relevant first or second-best solution will not be implementable by a loan scheme.□

Let us now introduce also the no-usury constraint (31). Following the line of argument used above, it can be shown that this constraint would be binding only in the presence of (30). Without the latter, the principal could in fact impose a tuition fee $\widetilde{\theta}^j(d^i_{FB})$ so high, that (a) $\pi^j_{S/PS}\left(k^j, y^j_{1FB}, y^j_{2FB}\right) >$

$\pi^j_{S/LF}\left(k^j, \widetilde{\theta}^j(d^i_{FB})\right)$ and (b) $y_{2FB}(m^j_2) \leq y_{1FB}(k^j) + \widetilde{\theta}^j(d^i_{FB})$ for every possible realization of m^j_2. For any $d^j \neq d^i_{FB}$, $\widetilde{\theta}^j(d^j)$ must be large enough for it to be true that $\pi^j_{S/PS}\left(k^j, y^j_{1FB}, y^j_{2FB}\right) > \pi^j_{S/LF}\left(k^j, \widetilde{\theta}^j(d^j)\right)$. That will not be feasible in the presence of (30). Therefore, the restriction on the size of tuition fees makes not only the credit participation, but also the no-usury constraint binding.